Derivation of Gauge Boson Masses from the Dynamics of Levy Flows

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Gauge bosons are fundamental fields that mediate the electroweak interaction of leptons and quarks. The underlying mechanism explaining how gauge bosons acquire mass is neither definitively settled nor universally accepted and several competing theories coexist. The prevailing paradigm is that boson masses arise as a result of coupling to a hypothetical scalar field called the Higgs boson. Within the current range of accelerator technology, compelling evidence for the Higgs boson is missing. We discuss in this paper a derivation of boson masses that bypasses the Higgs mechanism and is formulated on the basis of complexity theory. The key premise of our work is that the dynamics of the gauge field may be described as a stochastic process caused by the short range of electroweak interaction. It is found that, if this process is driven by Levy statistics, mass generation in the electroweak sector can be naturally accounted for. Theoretical predictions are shown to agree well with experimental data.

Key words: random walks and Levy flights, Yang-Mills theories, spontaneous symmetry breaking, electroweak model, gauge bosons

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1 Introduction and motivation

The electroweak model is a key component of the standard theory of particle physics. It successfully describes the coupling of leptons and quarks and integrates three massive gauge bosons responsible for the weak interaction (W^+, W^-, Z^0) with the massless photon of the electromagnetic interaction (γ). The existence of the four gauge bosons stems from a powerful principle of invariance called local gauge symmetry. An important consequence of this principle is that all four-gauge bosons must be massless. The mechanism through which the gauge triplet (W^+, W^-, Z^0) acquires mass while leaving the photon massless is called spontaneous symmetry breaking (SSB). As it is known, the prevailing paradigm of contemporary particle physics is that SSB results from interactions of all field quanta (including gauge bosons) with a hypothetical spin-0 field called Higgs boson. To date there is no conclusive experimental evidence for the Higgs, at least within the current energy range of particle accelerators. Searches for the Higgs boson are slated to continue at the Large Hadron Collider facility (LHC) over the next few years [1, 2].

Recently, a series of noteworthy efforts aiming at expanding the theoretical basis of the standard model have emerged. At the heart of these contributions lie the tools of chaos, nonlinear dynamics and fractals, as well as the novel mathematics of complexity. In the spirit of pioneering works developed by Beck [3], El Naschie [4] and following a similar method to [5], we introduce here a SSB mechanism that is formulated on the basis of complexity theory. The key premise of our model is that the dynamics of boson field may be described as a stochastic process originated in the short distance range of electroweak interaction. It is found that gauge boson masses can be dynamically generated on the assumption that this process is driven by Levy statistics.

Stochastic processes with Levy statistics are well-known prototypes of complex dynamics. From a phenomenological point of view, diffusive transport and random walks are remarkable transport mechanisms occurring in nature. Diffusion is the result of microscopic randomness and use of Laplacian operators to model it rests on the tacit assumption that randomness can be simulated as a Gaussian process. In recent years, a growing number of studies have shown the prevalence of non-Gaussian processes such as anomalous diffusion and Levy flights. To provide a more transparent connection between the physics of Levy flights and random walks, on the one hand, and gauge field theory, on the other, we shall use throughout the paper the term "flow" as a generic substitute for "flight" or "walk". The Levy flow of index $0 < \alpha \leq 2$ represents a stochastic transport process with Markov statistics, defined by long spatial steps that are distributed according to the power law $|x|^{-(1+\alpha)}$ [6, 7]. Mathematical modeling of Levy flows requires substitution of the Laplacian with fractal operators [6-8]. Levy flows have found numerous applications in science and engineering from the study of turbulence to the physics of plasma, molecular collisions and propagation through disordered media [9, 10]. As noted, motivation for using the dynamics of Levy flows in our work is based on the unavoidable regime of persistent fluctuations associated with the short range of electroweak interaction. Theoretical predictions are shown to agree well with experimental data.

The outline of the paper is as follows: section 2 introduces the concept of Levy flows. Section 3 connects the irregular dynamics of the gauge field to the evolution of Levy flows confined by a quartic potential. Masses of $W^{\pm}$ and Z^0 are retrieved in section 4. Next section computes two dynamic characteristics of the boson field flow, namely its Levy index and bifurcation time. A brief discussion on open questions and future challenges is detailed in section 6. Concluding remarks are presented in the last section.

2 Levy flows in steep potentials: a brief overview

To make the paper self-contained, we begin with a survey of the key notions and results developed in [9, 10]. The flow of a scalar wave in 1+1 dimensions through a randomly fluctuating medium with Levy statistics is described by the overdamped Langevin equation [10]

$$\frac{dx}{dt} = \frac{1}{\gamma_x}\left(-\frac{dU}{dx}\right) + Y_\alpha(t) \tag{1}$$

where γ_x represents the damping coefficient and $U \doteq U(x)$ is a generic anharmonic potential given by

$$U(x) = \lambda \frac{|x|^c}{c} \tag{2}$$

Here, $\lambda > 0$ determines the strength of the potential, $c \geq 2$ and $Y_\alpha(t)$ stands for stationary white noise with Levy index α. Following [10], we restrict the domain of α to the interval [1,2]. The evolution of the random flow may be alternatively studied with the help of the corresponding fractional Fokker-Planck equation

$$\frac{\partial \rho(x,t)}{\partial t} = \frac{1}{\gamma_x}\frac{\partial}{\partial x}\left[\frac{dU}{dx}\rho(x,t)\right] + D\frac{\partial^\alpha \rho(x,t)}{\partial |x|^\alpha} \tag{3}$$

in which D denotes the diffusion constant and $\rho(x,t)$ the probability density function of finding the center of the wave at location x and time t. Equations (1) and (3) may be brought to a dimensionless form by using the substitutions

$$x^0 = \frac{x}{x_0},\ t^0 = \frac{t}{t_0} \tag{4}$$

where x_0, t_0 are the space-time flow amplitudes defined according to

$$x_0 = \left(\frac{D\gamma_x}{\lambda}\right)^{\frac{1}{(c-2+\alpha)}} \qquad t_0 = \frac{x_0^\alpha}{D} \tag{5}$$

The probability density function satisfies the normalization condition

$$\int_{x_L}^{x_U} \rho(x,t)\, dx = 1 \tag{6}$$

where $x_{L(U)}$ are the lower and upper spatial bounds of the flow. A distinctive property of the free Levy flow ($U(x) = 0$) is that its variance diverges $\langle x^2(t)\rangle \to \infty$ for large $|x|$ [8-10]. In contrast, Levy flows confined by anharmonic potentials of the type (2) with $c > 2$ display two remarkable attributes:
a) the variance becomes finite for $c > 4 - \alpha$, that is

$$\langle x^2(t)\rangle < \infty \text{ if } c > 4 - \alpha \tag{7}$$

b) the probability density function $\rho(x,t)$ bifurcates from an initial mono-modal state to a stationary bimodal state. Figure 1 below illustrates this type of behavior for the case of a stationary quartic oscillator characterized by $c = 4$, $\alpha = 1$ and the probability density function

$$\rho_{st}(x) = \frac{1}{\pi(1 - ax^2 + x^4)} \tag{8}$$

where a plays the role of a control parameter (for additional details see [9, 10]).

We now proceed to give a physical interpretation to these general findings. From a field theoretic viewpoint, a free Levy flow is a highly delocalized process that corresponds to stochastic propagation of massless quanta. A relevant example for this type of process is the Maxwell field of stochastic electrodynamics whose photons carry the energy flow over an infinite range. In contrast, addition of a steep potential with power exponent higher than $c = 2$ confines the random energy flow to a bounded region of space. This translates into a well-defined transition from massless to massive field quanta. All non-abelian gauge theories that make up the standard model of particle physics are characterized by quartic self-interaction potentials ($c = 4$). As it may be anticipated from this discussion and according to (7), adding Levy noise to these theories is expected to generate massive boson states for any $\alpha > 0$.

It is known that the analog of probability density function in field theory is the concept of charge density [11]. It follows that bifurcation of probability density function, arisen from coupling the field to steep potentials, corresponds to physical splitting of the charge density flow at specific space locations. In field theoretic terms, bifurcation of the charge density may be viewed as a creation-annihilation event involving emission and absorption of particles at each interaction vertex [11-13].

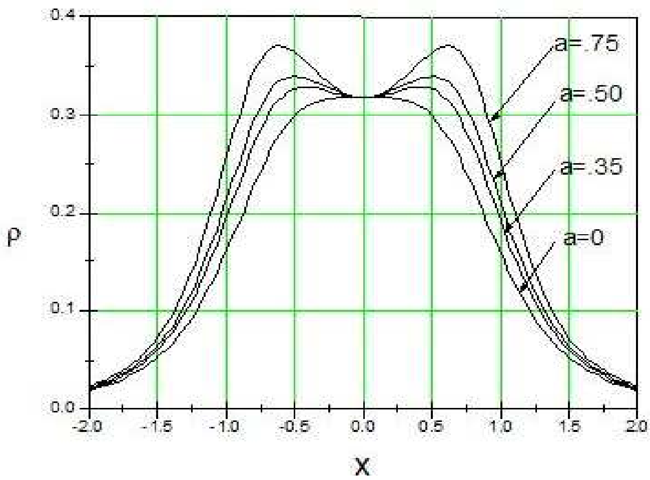

FIG. 1. Probability density function ρ of the stationary quartic oscillator for $\alpha = 1$ and for various values taken by the control parameter a.

3 Levy flows and gauge field theory

In this section we apply the general framework previously outlined to the dynamics of gauge fields operating in the electroweak model. To carry out this task we need to properly link the evolution of a generic Levy flow to the physics of stochastic gauge fields. We proceed with the following set of working assumptions:
a) Fluctuations of the gauge field are homogeneous random processes whose probability density function is a Levy stable law of index $1 \leq \alpha \leq 2$ [9,10].
b) Gauge fields are considered classical objects. Suppression of quantum attributes and transition to classical behavior is the signature of decoherence caused by steady exposure to random fluctuations [14, 15].

c) Gauge fields are homogeneous (space-independent). This approximation is consistent with the short range of electroweak interaction [16-18]. Without loss of generality and following [19, 20], we choose a representation where the temporal components of the gauge field take the simple form

$$A_0^1 = A;\ A_0^2 = 0;\ A_0^3 = -A \tag{9}$$

d) The electroweak interaction scale is set by the Fermi constant (G_F) where

$$G_F^{-\frac{1}{2}} \approx 293\,GeV \tag{10}$$

Under these circumstances, the gauge field Lagrangian contains a quartic self-interaction term written as [19, 20]

$$U_{self}(A) \sim g^2 \frac{A^4}{2} = 2g^2 \frac{A^4}{4} \tag{11}$$

in which g is the coupling charge. Two of the three massive bosons of the electroweak model, namely W^+, W^- carry both electromagnetic and weak isospin charges whereas Z^0 carries only the weak isospin charge. The photon (γ) is massless, carries no charge and does not couple to itself. On account of previous assumptions and comparing (11) to (2) leads to the observation that stochastic dynamics of the gauge field may be treated as a Levy flow confined by a quartic potential. The two processes may be matched with the straightforward prescription

$$\begin{aligned} |x| &\to A \\ \lambda &\to 2g^2 \end{aligned} \tag{12}$$

This section is divided in two parts. The first part deals with setting the correct formulation for time and field amplitudes based on dimensional analysis. The second part makes the connection between time amplitude and the observed decay width of gauge bosons.

3.1 Dimensional analysis

In "d"-dimensional space-time, the mass dimension of an arbitrary field is [13]

$$[A] = [M]^{\frac{d}{2}-1} \tag{13}$$

It follows that the gauge field is a scalar in 1+1 dimensions. Based on (4), its most natural representation is given by

$$A = \frac{m}{G_F^{-\frac{1}{2}}} \tag{14}$$

where m denotes the mass acquired as a result of transition from a free flow ($U_{self}(A) = 0$) to a flow confined by the self-interacting potential ($U_{self}(A) \neq 0$)[1]. By analogy with (5), the time amplitude of the gauge field may be introduced as

$$T_0 = \frac{A_0^\alpha}{D_A} \tag{15}$$

in which A_0 is the field amplitude and D_A the diffusion coefficient of the field flow. Let γ_A stand for the dissipation parameter associated with this flow. If we set the equivalent Boltzmann temperature of the flow to match the energy scale of the electroweak interaction (i.e., $kT = G_F^{-\frac{1}{2}}$), the fluctuation-dissipation theorem reads[2][8]

$$D_A \sim \frac{kT}{\gamma_A} = \frac{G_F^{-\frac{1}{2}}}{\gamma_A} \tag{16}$$

From (15) and (16) we conclude that D_A and γ_A have the following mass dimensions, respectively

$$\begin{aligned} [D_A] &= [M] \\ [\gamma_A] &= [M]^0 = scalar \end{aligned} \tag{17}$$

Comparison to (5) suggests that the most natural expression for the field amplitude is given by

$$A_0 = \left(\frac{D_A}{G_F^{-\frac{1}{2}}} \frac{\gamma_A}{2g^2} \right)^{\frac{1}{\alpha+2}} = \left(\frac{D_A}{G_F^{-\frac{1}{2}}} \frac{\gamma_A}{8\pi\alpha_{self}} \right)^{\frac{1}{\alpha+2}} \tag{18}$$

[1] This process bears resemblance to the Anderson localization of quantum waves in random potentials, a well-known phenomenon in condensed matter physics.

[2] We make here the tacit assumption that the fluctuation-dissipation theorem of equilibrium statistical mechanics is applicable to our context and an equivalent Boltzmann temperature of the Levy flow may be accordingly defined.

Here α_{self} represents the self-interaction strength ($\alpha_{self} = \frac{g^2}{4\pi}$). In light of the above discussion, α_{self} takes on two possible values:

$$\alpha_{self} = \alpha_2 \text{ for } Z^0 \qquad (19)$$
$$\alpha_{self} = \alpha_2 + \alpha_{EM} \text{ for } W^+, W^-$$

in which α_2 is the weak isospin strength and $\alpha_{EM} = \frac{e^2}{4\pi}$ is the fine structure constant. Addition of α_{EM} in (19) reflects the contribution of Coulomb interaction to the self-coupling process of the two charged vector bosons. Solving for m from (14) and (18) yields

$$m = \frac{G_F^{-\frac{1}{4}}}{2\sqrt{2\pi}}\sqrt{\frac{\gamma_A}{\alpha_{self} T_0}} \qquad (20)$$

The above relation may be used to determine the masses of $Z^0, W^\pm$ upon replacing α_{self} with (19). Further progress depends on specification of both dissipation parameter γ_A and time amplitude T_0. This is the object of the next paragraph.

3.2 Boson lifetime

Gauge bosons are narrow resonances with lifetimes on the order of $\tau_{W,Z} = 2.5 \times 10^{-25} sec.$ or, equivalently, with decay widths on the order of [16,17]

$$\Gamma_{W(Z)} = \left(\frac{1}{\tau_{W(Z)}}\right) \approx 2.6\, GeV \qquad (21)$$

The decay widths include all possible contributions from transitions where $W^\pm$ or Z^0 decay into observable leptons or quarks. The most natural ansatz is to assume

$$\tau_{W(Z)} = T_{0,W(Z)} \qquad (22)$$

where the spectrum of decay widths for the two bosons is taken from the literature [21]. The dissipation parameter γ_A is not directly accessible through experiment and it cannot be fixed by plausible arguments[3]. In what follows we are going to use it exclusively as normalization constant.

4 Boson masses

We are now ready to predict the masses of $W^\pm$ and Z^0 bosons based on (20). Using the following input parameters [17],[21]

$$\Gamma_W = 2.230\, GeV$$
$$\Gamma_Z = 2.487\, GeV$$
$$\alpha_2 = \frac{1}{29}$$
$$\alpha_{EM} = \frac{1}{128}$$

where α_{EM} is computed at the electroweak scale set by $G_F^{-\frac{1}{2}}$, we obtain

$$m_{W^\pm} = 78.4\, GeV$$
$$m_{Z^0} = 91.7\, GeV$$

if the dissipation parameter is normalized to $\gamma_A = 10 \approx \pi^2$. Since γ_A enters as a multiplicative constant in (20), a different choice for its numerical value preserves only the ratio of predicted boson masses, that is, the Weinberg angle.

5 Dynamic attributes of the field flow

It is instructive to compute the characteristic properties of the Levy flow for each of the $W^\pm$ and Z^0 bosons. To this end, we use (3) and start this section with the asymptotic expression of the stationary probability density function for large field values [10]

$$\rho_{st}(A) \approx \frac{sin(\frac{\pi\alpha}{2})\Gamma(\alpha)}{\pi A^{\alpha+3}}, A \gg 0 \qquad (23)$$

Invoking (6) and taking the lower limit of the normalization integral to match the field amplitude (14) yields, respectively

$$\int_{\frac{m_W}{G_F^{-\frac{1}{2}}}}^{\infty} \rho_{st}(A, \alpha_W)\, dA = 1 \qquad (24)$$

$$\int_{\frac{m_Z}{G_F^{-\frac{1}{2}}}}^{\infty} \rho_{st}(A, \alpha_Z)\, dA = 1 \qquad (25)$$

[3]This is because a) statistical physics of Levy flows is not a fully developed theory and b) the damping mechanism for electroweak fields subjected to Levy fluctuations cannot be put on the same footing with the damping experienced by gluons in high temperature QCD models.

It is found that the following pair of Levy indices solves (24) and (25)

$$\alpha_W = 1.956$$
$$\alpha_Z = 1.914$$

Lifetime of the gauge boson is defined as the interval elapsed from its creation to annihilation and accounts for the superposition of all possible decay events. In contrast, the bifurcation time marks the interval elapsed between two consecutive decay events consisting of emission and absorption of virtual quanta. The bifurcation time in a quartic potential is estimated by [10]

$$t_{bif} \approx \left[\frac{4\Gamma\left(\frac{3}{\alpha}\right)}{3(3-\alpha)\Gamma\left(\frac{1}{\alpha}\right)} \right]^{\frac{\alpha}{(2+\alpha)}} \tag{26}$$

from which we determine

$$t_W^{bif} = .810$$
$$t_Z^{bif} = .806$$

Thus, in original units, the actual bifurcation times are

$$t_{act,W}^{bif} = t_W^{bif} G_F^{\frac{1}{2}} = 2.765 \times 10^{-3}\, GeV^{-1}$$
$$t_{act,Z}^{bif} = t_Z^{bif} G_F^{\frac{1}{2}} = 2.751 \times 10^{-3}\, GeV^{-1}$$

6 Open questions

The approach we took in this paper is based on a series of simplifying assumptions regarding the dynamics of classical gauge fields exposed to steady Levy fluctuations. Needles to say, our work is far from being complete. Future studies are necessary to substantiate our findings in a more realistic context. The mass formula (20) is not derived from first principles and, as such, depends on parameters that are fixed by experiment. A key question that awaits clarification is related to the physical origin of the damping parameter whose value was chosen to properly match the observed masses of electroweak bosons. Furthermore, we believe that it is worthwhile to investigate if the SSB mechanism developed here can provide insights into the physics of mass generation in the QCD sector. As it is known, gluons are spin-1 bosons whose existence is determined by the principle of local gauge symmetry. Their vanishing masses comply with the unitarity requirement of quantum field theory. Under certain circumstances, it can be shown that gluons attain mass without violating unitarity. In this context it is of interest to explore if our framework is applicable to the following topics: a) dynamical generation of the off-diagonal gluon masses [22, 23], b) the emergence of massive gluons in radiative decays of heavy quarkonia systems [24] and c) numerical estimates of the glueball spectrum [25]. We briefly point out an interesting possibility regarding the last two topics. Let $\mu = 200\, MeV$ and $\alpha_3 = .1$ set the standard scale and strength of the strong interaction. To a first order approximation, we estimate the decay width for the massive gluon (Γ_g) to lie within

$$\mu \le \Gamma_g \le \Lambda_{\overline{MS}}$$

Here, we follow [22] and take $\Lambda_{\overline{MS}} \approx 500\, MeV$ to represent the scale set by the minimal subtraction scheme. Replacing these parameters in (20) and assuming that dissipation is unchanged by the transition from the electroweak theory to QCD, we derive

$$m_g = 384\, MeV \text{ if } \Gamma_g = \mu$$
$$m_g = 607\, MeV \text{ if } \Gamma_g = \Lambda_{\overline{MS}}$$

which correlate reasonably well with the range of gluon masses discussed in [24-25].

7 Concluding remarks

In summary, the goal of this paper was to give analytic evidence that gauge boson masses may be derived from the complex dynamics of Levy flows. It was shown how bosons attain mass as the result of flow localization in the self-interacting potential of gauge theory. The set of Levy indicies and bifurcation times were determined starting from the fractional Fokker-Planck equation and the density normalization condition. Table 1 reports the main results. It seems conceivable that the same SSB mechanism plays a role in dynamical mass generation of gluons as predicted by the physics of boson condensation.

Table 1: Summary of main results.

parameter	predicted value	experimental value
$m_W(GeV)$	78.40	80.46
$m_Z(GeV)$	91.70	91.19
α_W	1.956	-
α_Z	1.914	-
$t^{bif}_{W(Z)}$	.810 (.806)	-

References

[1] http://www.pparc.ac.uk/Rs/Fc/LHC.asp

[2] Kramer M. *Large Hadron Collider Phenomenology.* (IOP, 2004).

[3] Beck C. *Spatio-temporal chaos and vacuum fluctuations of quantized fields.* (World Scientific, Singapore, 2002).

[4] El Naschie M. The VAK of vacuum fluctuations, spontaneous self-organization and complexity theory interpretation of high energy particle physics and the mass spectrum. Chaos, Solitons and Fractals. **18**, 401 (2003). A review of E infinity theory and the mass spectrum of high energy particle physics. Chaos, Solitons and Fractals. **19**, 209 (2004).

[5] Goldfain E. Derivation of lepton masses from the chaotic regime of the linear σ-model. Chaos, Solitons and Fractals. **14**, 1331 (2002).

[6] Gnedenko B. V., Kolmogorov A. N. *Limit Distributions for Sums of Random Variables.* (Addison-Wesley, Reading, 1954).

[7] Metzler R., Klafter J. The random walk's guide to anomalous diffusion: a fractional dynamics approach. Phys. Reports. **339**, 1 (2000).

[8] West, B. J.; Bologna M.; Grigolini P. *Physics of fractal operators.* (Springer-Verlag, New York, 2003).

[9] Chechkin A. V., Klafter J., Gonchar V. Y., Metzler R., Tanatarov L. V. Bifurcation, bimodality, and finite variance in confined Levy flights. Phys. Rev. E. **67**, 010102 (2003).

[10] Chechkin A. V., Klafter J., Gonchar V. Y., Metzler R., Tanatarov L. V. Levy flights in a steep potential well. Stat. Phys. **115(5-6)**, 1505 (2004).

[11] Hatfield B. Quantum Field Theory of point particles and strings. *Frontiers in Physics.* (Westview Press, Oxford, vol.75, 1992).

[12] Cheng T. P.; Li L. F. *Gauge theory of elementary particle physics.* (Oxford University Press, New York, 1989).

[13] Ryder L. H. *Quantum Field Theory.* (Cambridge University Press, Cambridge, 1989).

[14] Zurek W. H. Phys. Today. **44**, no.10, 36 (1991). Zeh H. D. Phys. Lett. **A**, no. 172, 189 (1993).

[15] Elze H. T. at arXiv:quant-ph/9710063 v1; arXiv: hep-ph/9407377 v1.

[16] Donoghue J. F., Golowich E., Holstein B. R. *Dynamics of the Standard Model.* (Cambridge University Press, Cambridge, 1992).

[17] Kane G. *Modern Elementary Particle Physics.* (Addison-Wesley Publishing Company, New York, 1987).

[18] Kaku M. *Quantum Field Theory.* (Oxford University Press, New York, 1993).

[19] Savvidy G. K. The Yang-Mills classical mechanics as a Kolmogorov K-system. Phys. Lett. **130B**, no. 5, 303 (1983).

[20] Savvidy G. K. Classical and quantum mechanics of non-abelian gauge fields. Nucl. Phys. **B246**, 302 (1984).

[21] Abazov V. M. *et al.* Direct measurement of the W boson decay width. Phys. Rev. **D**, no. 66, 03008 (2002).

[22] Dudal D. *et al.* Analytic study of the off-diagonal mass generation for Yang-Mills theories in the maximal Abelian gauge. Phys. Rev. **D**, no. 70, 114038 (2004).

[23] Bornyakov V. G. et al. Phys. Lett. **B**, no. 559, 214 (2003).

[24] Field J. H. Phys. Rev. **D**, no. 66, 013013 (2002).

[25] Giacosa F.; Gutsche T.; Faessler A. at hep-ph/0408085. Phys. Rev. **C**, (to be published)].